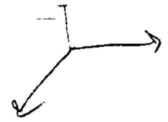
Quiz 8 Calc IV (Math 2443-002) 7/24/2002



1. Compute $\iint_S \mathbf{F}\cdot d\mathbf{S}$ for $\mathbf{F}(x,y,z)=\langle x^2y, xyz, z\rangle$ and where S is a disc of radius 2 centered on the z axis in the plane z=5, with upward orientation.

I $x(u,v) = u$ $\vec{r}(u,v) = \langle u, v, 5\rangle$

$y(u,v) = v$ $0 \le u \le$

$z(u,v) = 5$ $0 \le v \le$

II $\vec{F}(\vec{r}(u,v)) = \langle u^2v, uv5, 5\rangle$

III $\vec{r}_u = \langle 1,0,0\rangle$ $\vec{r}_u \times \vec{r}_v = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0-0)i - (0-0)j + \ldots$

$\vec{r}_v = \langle 0,1,0\rangle$ $= \langle 0,0,1\rangle$

IV $\iint_S \vec{F}\cdot d\vec{S} = \iint_S \langle u^2v, uv5, 5\rangle \cdot \langle 0,0,1\rangle \, dA$

V $\iint_S 0+0+5 \, dA = \iint_S 5 \, dA$

Great

$= \int_0^{2\pi}\int_0^2 5 r\,dr\,d\theta = \int_0^{2\pi} \frac{5r^2}{2}\Big|_0^2 d\theta = \int_0^{2\pi} 10\,d\theta = 10\theta\Big|_0^{2\pi}$

2. Use Stokes' Theorem to compute $\iint_S \text{curl}\,\mathbf{F}\cdot d\mathbf{S}$ for $\mathbf{F}(x,y,z)=\langle xy, y^2z, z\rangle$ and where S is a disc of radius 2 centered on the z axis in the plane z=5, with upward orientation.

$x(t) = 2\cos t$

$y(t) = 2\sin t$

$z(t) = 5$ $0 \le t \le 2\pi$

$\vec{r} = \langle 2\cos t, 2\sin t, 5\rangle$

$\vec{F}(\vec{r}(t)) = \langle 4\cos t\sin t, 20\sin^2 t, 5\rangle$

Well done

$\vec{r}'(t) = \langle -2\sin t, 2\cos t, 0\rangle$

$\int_0^{2\pi} \langle 4\cos t\sin t, 20\sin^2 t, 5\rangle \cdot \langle -2\sin t, 2\cos t, 0\rangle \, dt$

$= \int_0^{2\pi} -8\cos t\sin^2 t + 40\cos t\sin^2 t \, dt$

$= \int_0^{2\pi} 32\cos t(\sin t)^2 \, dt$

$u = \sin t$, $du = \cos t\,dt$, $dt = \frac{du}{\cos t}$

$= \int 32u^2\,du = \frac{32u^3}{3}\Big|_0^{2\pi} = \frac{32\sin(t)^3}{3}\Big|_0^{2\pi} = \boxed{0}$