Exam 2 Calc 1 7/15/2004

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Prove the Sum Rule for derivatives.

2. Find the exact x values where the absolute minimum and maximum values of the function $f(x) = x^3 - 6x + 5$ occur on the interval $[0, 2]$.

3. A curve is given parametrically by $x(t) = 2 - t^2$, $y(t) = 4t^4 + 1$.

a) When $t = 1$, what are the coordinates of the corresponding point?

b) Find the coordinates of all points on the curve where the tangent line is horizontal.

4. If $f(2) = 5$, $f'(2) = 1$, $g(2) = -3$, and $g'(2) = 7$, what are:

a) $(f - g)'(2)$?

b) $(f \times g)'(2)$?

c) $\left(\frac{f}{g}\right)'(2)$?

5. Find $\lim_{x \to 0} \frac{\sin(3x)}{x}$. Be clear in your justification!

6. Prove the Quotient Rule for derivatives [you may feel free to use the Product Rule if you like].

7. Find all points on the curve defined by the equation $y^2 + 2xy + x^2 + x - y = 0$ where the tangent line is vertical.

8. Bunny is a calculus student at a large state university and she's having some trouble. She says "So this second derivative test thingy totally confuses me. First, I can't get it straight what it means when it's plus or minus, 'cause it seems kinda backwards or something. But I'm more confused than just that, too, 'cause I don't think it makes sense anyway. I mean, if there was a spot on the graph where it was a minimum, like the shape at the bottom of a parabola or whatever, then wouldn't *all* the derivatives be zero, like the first derivative and the second derivative too, and all the rest? Because it's not changing there, right, like it's not up or down at all? So *all* the derivatives would be zero and they wouldn't have plus or minuses at all? And I think I better figure this out, or else my grade might not be good enough to pledge a good sorority!"

Explain clearly to Bunny what's up with the second derivative at a local maximum or minimum point, and whether it can (or must) be zero at such a point.

9. Find a function of the form $f(x) = ax^2 + bx + c$ (for some values of the constants a, b, and c) which passes through both the origin and the point (2,3) and has a tangent at (2,3) with a slope of 5.

10. Suppose that $f(x)$ is a function with $f(x) > 0$ for all values of x. Let $g(x) = 1/f(x)$.
    a) If f has a local maximum at x_1, what can you say about g? How do you know?

    b) If f is concave up at x_2, what can you say about g? How do you know?

Extra Credit (5 points possible):
What can you say about the derivative of arcsinh x?