Problem Set 4 Calculus 1 Due 7/8/04

You are encouraged to work in groups of two to four on this assignment and make a single group submission. Each problem is worth 5 points. For full credit indicate clearly how you reached your answer. All work must be legible and submitted on clean paper without ragged edges.

1. The curve given by the implicit equation $x^3 + y^3 = 6xy$ is called the folium of Descartes.

a) Find an expression for the slope of the line tangent to this curve at the point (x,y).

b) Write an equation for the line tangent to this curve at the point (3,3).

c) Find the coordinates of all points on the curve where the tangent line is horizontal.

d) Find the coordinates of all points on the curve where the tangent line is vertical.

2. Consider the curve given by the implicit equation $x^2 + xy + y^2 = 1$.

a) Find an expression for the slope of the line tangent to this curve at the point (x,y).

b) Write an expression for the line tangent to this curve at the point (1,-1).

c) Find the coordinates of all points on the curve where the tangent line is horizontal.

d) Find the coordinates of all points on the curve where the tangent line is vertical.

3. Consider the curve given by the parametric equations $x(t) = 2t^2 - t$, $y(t) = 3t^3 - t$.

a) When $t = 2$, what are the coordinates of the corresponding point?

b) Find the coordinates of all points where the curve crosses the x-axis.

c) Find an expression for the slope of the line tangent to this curve at time t.

d) Find the coordinates of all points on the curve where the tangent line is vertical.

4. Curves with parametric equations of the form $x(t) = a\cos(t)$, $y(t) = a\sin(t)$, for various values of the constant a, have a familiar shape.

a) What role does the value of a play?

b) Which values of t correspond to the portion of such a curve which lies in the first quadrant?

c) Find an expression for the slope of the line tangent to this curve at time t.