Exam 1   Calc 1   6/22/2006

Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. State the definition of the derivative of the function f(x) at the point $x = a$.

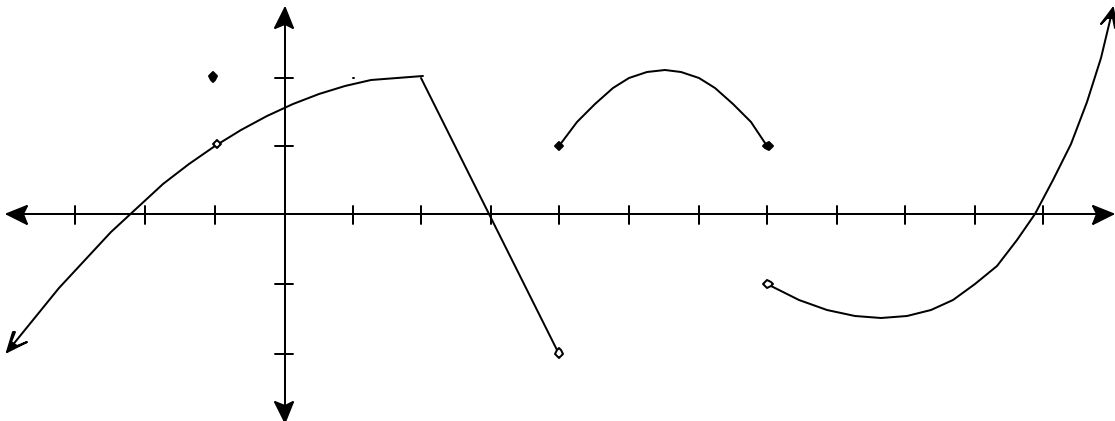Use the graph of f(x) at the bottom of the page for problems 2 and 3:

   a) What is f(-1)?

   b) What is $\lim\limits_{x \to -1} f(x)$?

   c) What is $\lim\limits_{x \to 2^+} f(x)$?

3. a) For which value(s) of x is f(x) not continuous? Why?

   b) For which value(s) of x is f(x) not differentiable? Why?

4. Estimate $\displaystyle\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$ numerically.

5. Evaluate $\displaystyle\lim_{x\to\infty}\frac{\sqrt{9x^4-x^2}}{25-x^2}$ exactly.

6. If $f(x) = \sqrt{3x+1}$, use the definition of the derivative to find $f'(2)$.

7. Evaluate the limit $\lim\limits_{x \to 3} \dfrac{x^2}{2x-1}$ and justify each step by indicating the appropriate limit law(s) from the list below.

Algebraic Limit Properties

Let c be a constant. Then as long as $\lim\limits_{x \to a} f(x)$ and $\lim\limits_{x \to a} g(x)$ exist,

Constant Rule for Limits:	$\lim\limits_{x \to a} c = c$
Rule X for Limits:	$\lim\limits_{x \to a} x = a$
Sum Rule for Limits:	$\lim\limits_{x \to a}\left[f(x) + g(x) \right] = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$
Difference Rule for Limits:	$\lim\limits_{x \to a}\left[f(x) - g(x) \right] = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$
Constant Multiple Rule for Limits:	$\lim\limits_{x \to a}\left[c \cdot f(x) \right] = c \cdot \lim\limits_{x \to a} f(x)$
Product Rule for Limits:	$\lim\limits_{x \to a}\left[f(x) \cdot g(x) \right] = \lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)$
Quotient Rule for Limits:	$\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)}$ as long as $\lim\limits_{x \to a} g(x) \neq 0$.
Power Rule for Limits:	$\lim\limits_{x \to a}\left[f(x) \right]^n = \left[\lim\limits_{x \to a} f(x) \right]^n$

8. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod, we had this test, and it was so messed up. In high school math was always just working out problems, but now there are these problems where, like, they want you to say *why* something is what it is. I thought that was philosophy or something. But so there was this one question on our test about whether this equation $e^x = 2 - x$ had a solution, and when the grad student who runs our class handed the test back, he was saying all this stuff about how we were supposed to give some important reason to explain it. I don't know how to explain it, I just tried lots of things to solve it and couldn't!"

Explain to Bunny, as clearly as possible, either how she might assure that there is a solution to this equation, or how she could show that no solution exists.

9. Let $f(x) = mx + b$, where m and b are constants. Use the definition of the derivative to find $f'(a)$.

10. Suppose that g(x) = f(-x). What connection would you expect between g′ and f′, and why?

Extra Credit (5 points possible):

Jon drives 100 miles in 100 minutes. Is there a time, other than the instant he starts or the instant he finishes, when he has gone exactly as far as the number of minutes that have elapsed? Why?