1. Determine whether the function $y = \sin t$ is a solution to the differential equation $\frac{d^2 y}{dt^2} + y = \sin t$.

2. State the definition of a separable differential equation.
3. Sketch the phase line for the differential equation
\[ \frac{dy}{dt} = f(y) \] if \( f(y) \) has the graph shown:

4. Find a general solution to the differential equation
\[ \frac{dy}{dt} = t + ty^2. \]
5. Use Euler’s method with step size $\Delta t = 0.5$ to approximate $y(1)$ to the nearest hundredth for a solution $y$ to the differential equation $\frac{dy}{dt} = 2y + 1$ subject to the initial condition $y(0) = 3$. 
6. Find a general solution to the differential equation \( \frac{dy}{dt} = \frac{y}{t} + 4 \).
7. Suppose that $\frac{dy}{dt} = f(y)$ is a differential equation satisfying the hypotheses of our existence and uniqueness theorems. Further suppose that $y_1(t) = 0$, $y_2(t) = 20$, and $y_3(t) = 30$ are all solutions for all $t$. If you’re seeking a solution satisfying the initial condition $y(0) = 5$, what can you conclude about that solution?
8. Find the power series expansion for the general solution up to degree four to the differential equation \( \frac{d^2y}{dt^2} + y = \sin t \).
9. **Sketch the bifurcation diagram** for the differential equation \( \frac{dy}{dt} = y^3 + \alpha y^2 \). Include direction arrows on the phase lines and make clear the exact \( \alpha \) values where bifurcations occur.
10. For what value(s) of the parameter $r$ is it possible to find explicit formulas (without integrals) for the solutions to $\frac{dy}{dt} = t^r y + 4$?