Each problem is worth 10 points. For full credit provide complete justification for your answers.

1. Derive the formula \( \int x e^x \, dx = x e^x - e^x + C \).

2. Give the form for a partial fractions decomposition of

\[
\int \frac{2(x^4 + 1)}{(x - 2)(x^2 - 1)^2 (x^2 + 2)^2} \, dx,
\]

or explain why one does not exist.
3. Suppose you’re approximating \( \int_{3}^{7} f(x) \, dx \), and that the left-hand approximation with 10 subdivisions is 1.7034, the right-hand approximation with 10 subdivisions is 1.9282, and the midpoint approximation with 10 subdivisions is 1.8566. What are the trapezoidal and Simpson’s approximations with 10 subdivisions for this integral?

4. Evaluate \( \int \tan^4 \theta \, d\theta \).
5. Evaluate $\int_{8}^{\infty} \frac{dx}{\sqrt[3]{x}}$.

6. Evaluate $\int x \arctan(6x) \, dx$.
7. Bunny is a Calculus student at Enormous State University, and she’s having some trouble. Bunny says “Ohmygod! We were doing our homework last night, and there was this one problem, like integrating tangent of $e$ to the $x$, right? And the table of integrals has tan $x$, but can you just use that with $e$ to the $x$ where they have the $x$? It seems too easy.”

Help Bunny by explaining whether such an approach works. You do not need to actually work the integral out, but explain to Bunny whether her plan is valid.
8. Evaluate \( \int \frac{x^4}{1+x^2} \, dx \).
9. Evaluate \( \int \frac{1}{x^3 - 6x^2} \, dx \).
10. Derive the formula $\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx$ (for $n \neq 1$), line 43 from the table of integrals.
Extra Credit (5 points possible):

Derive the formula \( \int \sqrt{2ax - x^2} \, dx = \frac{x - a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x - a}{a} \right) + C \) (for \( a > 0 \)),

line 97 from the table of integrals.