1. (a) State the definition of a topology.

(b) Show that \( \emptyset \) satisfies the definition of a topology, or explain why it doesn’t.
2. (a) State the (topological) definition of continuity.

(b) Give an example of a function which is \( \mathcal{H} \rightarrow \mathcal{H} \) continuous but not \( \mathcal{U} \rightarrow \mathcal{U} \) continuous, or explain why it can’t be done.
3. (a) State the (topological) definition of a closed set.

(b) Give an example of a set which is closed in $\mathbb{R}$ with the $C$ topology.
4. (a) Given a function $f : A \rightarrow B$ and a set $V \subseteq B$, state the definition of $f^{-1}(V)$

(b) Give an example to show it can happen that $f^{-1}(f(U)) \neq U$. 
5. Let $B = \{(a, +\infty) : a \in \mathbb{Z}\}$.

(a) Is $B$ a base for a topology on $\mathbb{R}$?

(b) Is $B$ a base for the $C$ topology on $\mathbb{R}$?
6. Show that the composition of homeomorphisms is a homeomorphism. Feel free to note the portions that were taken care of in Foundations, but provide details on those that were not.
7. Let $\Lambda = \mathbb{Z}^+$ and for each $i \in \Lambda$, let $X_i = \mathbb{R}$ and let $\mathcal{T}_i = \mathcal{B}$. Which of the following are open subsets of the product space $\times\{X_i : i \in \Lambda\}$? If a set is not open, explain why it is not.

(a) $\times\{U_i : i \in \Lambda\}$, where $U_i = (0, 1)$ for each $i \in \Lambda$.

(b) $\times\{U_i : i \in \Lambda\}$, where $U_i = (0, 1)$ if $i$ is an odd integer and $\mathbb{R}$ if $i$ is an even integer.

(c) $\times\{U_i : i \in \Lambda\}$, where $U_1 = (0, 1)$ and $U_i = \mathbb{R}$ otherwise.
A. The collection $\mathcal{B} = \{x : x \in \mathbb{R}\}$ is a base for the usual topology on $\mathbb{R}$.
\[\Box\text{B. Let } (X, \mathcal{T}) \text{ be a topological space with } A \subseteq X \text{ and } U \subseteq A. \text{ The set } U \text{ is } \mathcal{T}\text{-closed iff } U = W \cap A \text{ for some } \mathcal{T}\text{–closed set } W.\]
C. Let $(X, \mathcal{T})$ and $(Y, \mathcal{S})$ be topological spaces. If $A$ and $B$ are closed subsets of $X$ and $Y$ respectively, then $A \times B$ is a closed subset of $X \times Y$. 
D. Let $(X, \mathcal{T})$ be a topological space. Let $A, B \subseteq X$.

(a) Prove that $(A \cap B)' \subseteq A' \cap B'$.

(b) Give an example to show that $(A \cap B)' \supseteq A' \cap B'$ does not hold.
E. Let $A = [0, 1) \cup (2, 3]$ be a subset of $(\mathbb{R}, \mathcal{H})$.

(a) Find $\text{Int}(A)$, and justify your answer well.

(b) Find $\text{Cl}(A)$, and justify your answer well.