You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. [Baker 5.2.6] Let $X$ be a connected topological space with $a, b \in X$ and let $f : X \to \mathbb{R}$ be a continuous function (where $\mathbb{R}$ has the usual topology). Prove that if $y$ is any number between $f(a)$ and $f(b)$, then there is an element $x \in X$ such that $f(x) = y$. (This is a slightly stronger version of the Intermediate Value Theorem.)

2. [Baker 5.2.7] Complete the proof of Theorem 5.2.1.

3. [Baker 5.2.13] Determine if the Intermediate Value Theorem holds if the $U$-relative topology on $[a, b]$ is replaced by the $H$-relative topology.

4. [Baker 5.2.21] Let $X = \{a, b, c\}$ and $\mathcal{T} = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}\}$. Find all cut points of $(X, \mathcal{T})$.

5. [Baker 5.2.22] Let $X$ and $Y$ be topological spaces and let $f : X \to Y$ be a homeomorphism. If $x$ is a cut point of $X$, then $f(x)$ is a cut point of $Y$.

6. [Baker 5.2.23] In the previous statement, determine if the requirement that $f$ be a homeomorphism can be replaced by the condition that $f$ be a continuous function.