Exam 2 In-class Portion   Differential Equations   3/17/06

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. Determine whether $x(t) = -e^{-2t}\sin 3t$, $y(t) = e^{-2t}\cos 3t$ is a solution to the system of differential equations

$$\begin{aligned}\frac{dx}{dt} &= -2x - 3y \\ \frac{dy}{dt} &= 3x - 2y\end{aligned}.$$

2. Find all equilibrium points of the predator-prey system

$$\begin{aligned}\frac{dR}{dt} &= 2R\left(1 - \frac{R}{2.5}\right) - 1.5RF \\ \frac{dF}{dt} &= -F + 0.8RF\end{aligned}$$

3. Find a general solution to the differential equation $y'' - y' - 12y = 0$.

4. a) Find a general solution to the partially decoupled system

$$\frac{dx}{dt} = 3x + 2y$$

$$\frac{dy}{dt} = -2y$$

b) Find a particular solution satisfying the initial condition $(x_0, y_0) = (5,3)$.

5. How do you know that Laplace transforms are linear, i.e. that $\mathscr{L}[a \cdot f(x) + b \cdot g(x)] = a\ \mathscr{L}[f(x)] + b\ \mathscr{L}[g(x)]$ for any functions f and g whose Laplace transforms exist?

Exam 2 Take-home Portion Differential Equations 3/17/06

Each problem is worth 10 points. You may freely consult our textbook or any notes you generated prior to receiving this exam, but may **not** consult with any living being directly or indirectly, nor outside resources of any sort (except of course spreadsheets or software not involving internet access).

6. Use Laplace transforms to find a solution to the differential equation $\frac{dy}{dt} = e^{3t} + 5y$ subject to the initial condition $y(0) = D$.

7. Consider the system of differential equations for two populations:

$$\frac{dx}{dt} = 8x - 2x^2 - 4xy$$

$$\frac{dy}{dt} = 9y - 5xy - 3y^2$$

a) Use Euler's method with $\Delta t = 0.05$ to approximate $x(3)$ and $y(3)$ if $x(0) = 0.7$ and $y(0) = 2.6$.

b) Use Euler's method with $\Delta t = 0.05$ to approximate $x(3)$ and $y(3)$ if $x(0) = 0.5$ and $y(0) = 2.6$.

c) Use Euler's method with $\Delta t = 0.05$ to approximate $x(3)$ and $y(3)$ if $x(0) = 0.86$ and $y(0) = 1.57$.

d) Comment on the equilibria of this system. Do they appear to be sources, sinks, or nodes?

8. Do problem #14 from §2.4 in the text.

9. Find the Laplace transform of the function $blip_{a,b}(t) = \begin{cases} 0, & \text{if } t \in (-\infty, a) \\ 1, & \text{if } t \in [a,b) \\ 0, & \text{if } t \in [b,\infty) \end{cases}$ **without** computing any integrals – use only facts we've already established in class.

10. If you know that $y(t)$ has Laplace transform $Y(s)$, with some domain $S \subseteq \mathbb{R}$, what can you say about the Laplace transform of $t \cdot y(t)$?