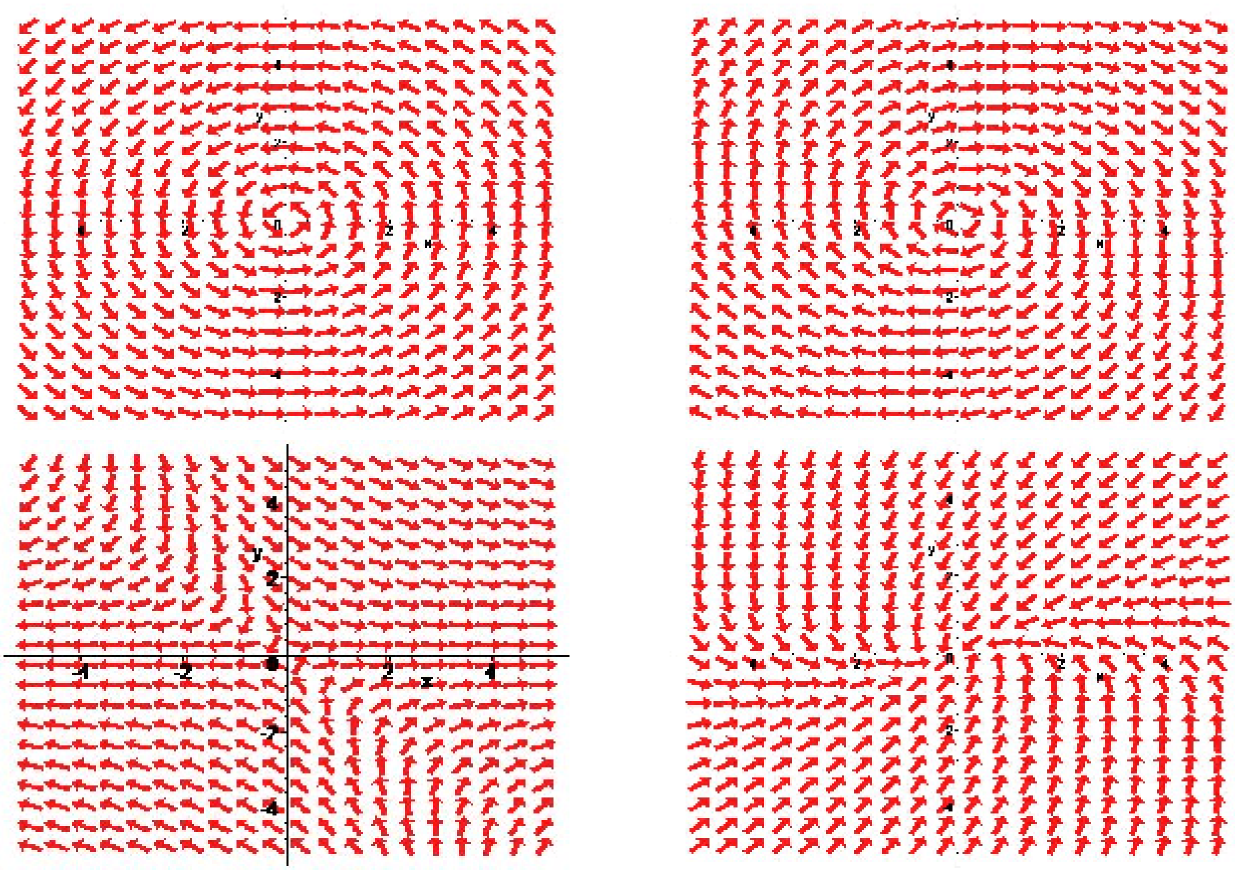
Exam 3　　Differential Equations　　4/14/06

Each problem is worth 10 points. For full credit indicate clearly how you reached your answer.

1. If a planar system of differential equations has eigenvalues $\lambda_1 = 3$, $\lambda_2 = -2$ and associated eigenvectors $\mathbf{v}_1 = (1,0)$ and $\mathbf{v}_2 = (3,-1)$, write a general solution to the system.

2. Classify each of the following planar systems' equilibria as sources, sinks, saddles, centers, spiral sources, or spiral sinks.



3. Find all eigenvalues of the system of differential equations $\begin{aligned} \frac{dx}{dt} &= -2x - 3y \\ \frac{dy}{dt} &= \quad 3x - 2y \end{aligned}$.

4. Given that the eigenvalues of the system $\begin{aligned} \frac{dx}{dt} &= 3x + 2y \\ \frac{dy}{dt} &= -2y \end{aligned}$ are $\lambda_1 = 3$ and $\lambda_2 = -2$, find the associated eigenvectors.

5. Express the eigenvalues of a planar system with coefficient matrix $A = \begin{pmatrix} 0 & 1 \\ -q & -p \end{pmatrix}$.

6. Given that a planar system with coefficient matrix $\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$ has

$$\mathbf{Y} = e^t \begin{pmatrix} \cos t - \sin t \\ 2\cos t \end{pmatrix} + i\, e^t \begin{pmatrix} \cos t + \sin t \\ 2\sin t \end{pmatrix}$$

As a solution, find a particular solution meeting the initial condition $\mathbf{Y}_0 = (2,0)$.

7. Prove that if λ is an eigenvalue of a matrix **A**, with corresponding eigenvector **v**, then $\mathbf{Y} = e^{\lambda t}\mathbf{v}$ is a solution to $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$.

8. Find an eigenvalue and the corresponding eigenvector for the matrix $\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$.

9. For what real values of a and b will the planar system $\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} a & b \\ b & 0 \end{pmatrix}\mathbf{Y}$ have a center?

10. Suppose the matrix **A** with real entries has complex eigenvalues $\lambda = \alpha + i\beta$ and $\overline{\lambda} = \alpha - i\beta$. Suppose also that $\mathbf{Y}_0 = (x_1 + iy_1, x_2 + iy_2)$ is an eigenvector for the eigenvalue λ. Show that $\overline{\mathbf{Y}_0} = (x_1 - iy_1, x_2 - iy_2)$ is an eigenvector for the eigenvalue $\overline{\lambda}$. In other words, the complex conjugate of an eigenvector for λ is an eigenvector for $\overline{\lambda}$. [Blanchard et al. 2^{nd} p. 299]