

Homework 3 Foundations 2/6/08

1. Suppose that $a, b \in \mathbb{R}$. If $a < b$, then $a < \frac{a+b}{2} < b$.
2. Suppose that $a, b \in \mathbb{R}$. If $a < b$ then $a^2 < b^2$.
3. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $a < b \Leftrightarrow a^2 < b^2$.
4. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $a < b \Leftrightarrow \sqrt{a} < \sqrt{b}$
5. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $\sqrt{ab} \leq \frac{a+b}{2}$.
6. Suppose that $a, b \in \mathbb{R}$. If $a, b > 0$, then $\sqrt{a^2 + b^2} \leq a + b$.
7. Suppose that $a, b \in \mathbb{R}$. Then $|a - b| \geq |a| - |b|$.
8. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $a + c < b + d$.
9. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $a - c < b - d$.
10. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $ac < bd$.
11. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$ and $b, c > 0$. Then $ac < bd$.
12. Suppose that $a, b, c, d \in \mathbb{R}$, with $a < b$ and $c < d$. Then $\frac{a}{c} < \frac{b}{d}$.
13. Suppose that $a, b \in \mathbb{R}$, with $a < b$ and $a, b > 0$. Then $\forall n \in \mathbb{N}$, $a^n < b^n$.
14. Suppose that $a, b \in \mathbb{R}$. If $a^2 = b^2$, then $a = b$.
15. Suppose that r is a real number. Then $r^2 \geq r$ and $\frac{1}{r^2} \leq \frac{1}{r}$.
16. Suppose that r is a real number and $r \geq 1$. Then $r^2 \geq r$ and $\frac{1}{r^2} \leq \frac{1}{r}$.