**Problem Set 10 Foundations Due 4/23/2008**

Each problem is worth 5 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor. Submissions must be on clean paper with no ragged edges.

- Given a set $A$, define $S(A)$ to be the set $A \cup \{A\}$.
- Define $N$ to be the set containing $\varnothing$ and such that $\forall x \in N$, $S(x) \in N$.

1. Write $S(\varnothing)$, $S(S(\varnothing))$, and $S(S(S(\varnothing)))$ explicitly. How many elements does each of these have?

2. Show that for any set $A$, $S(A) \neq \varnothing$.

3. Define a relation $\approx$ on the collection of all sets by $\approx\, = \{(A,B) \mid \exists f{:}A \to B \text{ a bijection}\}$. Show that $\approx$ is an equivalence relation.

- Define $\mathbb{N}$ to be the set of equivalence classes (under the equivalence relation specified in problem 3) whose representatives are members of the set $N$ defined above. It is customary to refer to the equivalence class of $\varnothing$ as 0, and similarly to label $[S(0)]$ as 1, $2 \equiv [S(1)]$, $3 \equiv [S(2)]$, $4 \equiv [S(3)]$, etc.

4. Show that $2 + 2 = 4$.