Midterm Exam **Set Theory & Topology** **3/4/16**

Do questions 1 through 7 and pick three of the remaining (lettered) questions for grading (check boxes of those you want graded or I roll dice). Each problem is worth 10 points. Show good justification for full credit. Dont panic.

1. (a) State the definition of a topology.

   (b) Show that $\mathscr{D}$ satisfies the definition of a topology, or explain why it doesn't.

2. (a) State the (topological) definition of continuity.

(b) Give an example of a function which is $\mathscr{H} - \mathscr{H}$ continuous but not $\mathscr{U} - \mathscr{U}$ continuous, or explain why it can't be done.

3. (a) State the (topological) definition of a closed set.

(b) Give an example of a set which is closed in $\mathbb{R}$ with the $\mathscr{C}$ topology.

4. (a) Given a function $f : A \to B$ and a set $V \subseteq B$, state the definition of $f^{-1}(V)$

(b) Give an example to show it can happen that $f^{-1}(f(U)) \neq U$.

5. Let $B = \{(a, +\infty) : a \in \mathbb{Z}\}$.

   (a) Is B a base for a topology on $\mathbb{R}$?

   (b) Is B a base for the $\mathscr{C}$ topology on $\mathbb{R}$?

6. Show that the composition of homeomorphisms is a homeomorphism. Feel free to note the portions that were taken care of in Foundations, but provide details on those that were not.

7. Let $\Lambda = \mathbb{Z}^+$ and for each $i \in \Lambda$, let $X_i = \mathbb{R}$ and let $\mathscr{T}_i = \mathscr{U}$. Which of the following are open subsets of the product space $\times\{X_i : i \in \Lambda\}$? If a set is not open, explain why it is not.

(a) $\times\{U_i : i \in \Lambda\}$, where $U_i = (0, 1)$ for each $i \in \Lambda$.

(b) $\times\{U_i : i \in \Lambda\}$, where $U_i = (0, 1)$ is i is an odd integer and $\mathbb{R}$ if i is an even integer.

(c) $\times\{U_i : i \in \Lambda\}$, where $U_1 = (0, 1)$ and $U_i = \mathbb{R}$ otherwise.

□ A. The collection $\mathscr{B} = \{\{x\} : x \in \mathbb{R}\}$ is a base for the usual topology on $\mathbb{R}$.

□ B. Let $(X, \mathscr{T})$ be a topological space with $A \subseteq X$ and $U \subseteq A$. The set U is $\mathscr{T}$-closed iff $U = W \cap A$ for some $\mathscr{T}$−closed set W.

□ C. Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces. If A and B are closed subsets of X and Y respectively, then $A \times B$ is a closed subset of $X \times Y$.

□ D. Let $(X, \mathscr{T})$ be a topological space. Let $A, B \subseteq X$.

(a) Prove that $(A \cap B)' \subseteq A' \cap B'$.

(b) Give an example to show that $(A \cap B)' \supseteq A' \cap B'$ does not hold.

□ E. Let $A = [0, 1) \cup (2, 3]$ be a subset of $(\mathbb{R}, \mathscr{H})$.

(a) Find $\text{Int}(A)$, and justify your answer well.

(b) Find $\text{Cl}(A)$, and justify your answer well.