Problem Set 2 **Set Theory & Topology** **Due 1/29/16**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. If A and B are sets and $A = B$, then $A - B = \emptyset$.
2. If A and B are sets and $A - B = \emptyset$, then $A = B$.
3. If both A and B are the empty set, then $A \times B = \emptyset$.
4. If A and B are sets and $A \times B = \emptyset$, then $A = \emptyset$ and $B = \emptyset$.
5. If A, B, and C are sets, then $A - (B - C) = (A - B) - C$.
6. If A and B are subsets of X and $A \cap B \neq \emptyset$, then $B \nsubseteq X - A$.
7. Let $\{A_\alpha : \alpha \in \Lambda\}$ be an indexed collection of sets. If $\bigcap\{A_\alpha : \alpha \in \Lambda\} = \emptyset$, then for any distinct α and β in Λ, $A_\alpha \cap A_\beta = \emptyset$.
8. Let $\{A_\alpha : \alpha \in \Lambda\}$ be an indexed collection of sets. If for any distinct α and β in Λ $A_\alpha \cap A_\beta = \emptyset$, then $\bigcap\{A_\alpha : \alpha \in \Lambda\} = \emptyset$.
9. If $f : X \to Y$ is a function and $f(x_1) = f(x_2)$, then $x_1 = x_2$.
10. If $f : X \to Y$ is a one-to-one function and $f(x_1) = f(x_2)$, then $x_1 = x_2$.
11. If $f : X \to Y$ is a function and V is a nonempty subset of Y, then $f^{-1}(V)$ is a nonempty subset of X.
12. If $f : X \to Y$ is an onto function and V is a nonempty subset of Y, then $f^{-1}(V)$ is a nonempty subset of X.
13. The inverse of the inverse of a one-to-one onto function is the original function.
14. Let $f : X \to Y$ be a function and let A and B be subsets of Y. If $f^{-1}(A) = f^{-1}(B)$, then $A = B$.
15. If $f : X \to Y$ is a function, then $f(X) = Y$.
16. If $f : X \to Y$ is onto, then $f(X) = Y$.
17. Inverse images of sets are only defined for one-to-one functions.
18. If $f : X \to Y$ is a function, then $f^{-1}(Y) = X$.
19. If $f : X \to Y$ is a function and U and V are subsets of X, then $f(U \cap V) = f(U) \cap f(V)$.
20. If $f : X \to Y$ is a function and U and V are subsets of X, then $f(U \cap V) \subseteq f(U) \cap f(V)$.