Problem Set 3 **Set Theory & Topology** **Due 2/15/16**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. Does the set $\{(a, +\infty) | a \in \mathbb{R}\}$ form a topology on $\mathbb{R}$?

2. [Baker 2.2.12] Determine if the function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} 2 & \text{if } x \geq 1 \\ -2 & \text{if } x < 1 \end{cases}$$

   is

   (a) $\mathscr{U} - \mathscr{U}$ continuous

   (b) $\mathscr{U} - \mathscr{H}$ continuous

   (c) $\mathscr{U} - \mathscr{C}$ continuous

   (d) $\mathscr{H} - \mathscr{U}$ continuous

   (e) $\mathscr{H} - \mathscr{H}$ continuous

   (f) $\mathscr{C} - \mathscr{H}$ continuous

   (g) $\mathscr{C} - \mathscr{C}$ continuous.

3. [Baker 2.3.13] Let U be a closed set and let V be an open set in a topological space. Show that $U - V$ is closed and that $V - U$ is open.

4. [Baker 2.3.14] Let A and B be subsets of a topological space $(X, \mathscr{T})$. Show that $(X - \text{Cl}(A)) \cup (X - \text{Cl}(B)) \subseteq X - \text{Cl}(A \cap B)$. Find an example that shows these sets are not in general equal.

5. [Baker 2.3.15] Let A and B be subsets of a topological space $(X, \mathscr{T})$. Show that

$$X - \text{Cl}(A \cup B) = (X - \text{Cl}(A)) \cap (X - \text{Cl}(B)).$$

6. [Baker 2.4.10] Show that if A is a subset of a topological space, then $\text{Int}(\text{Int}(A)) = \text{Int}(A)$.