Problem Set 4 **Set Theory & Topology** **Due 2/10/16**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. The empty set is a closed subset of $\mathbb{R}$ regardless of the topology on $\mathbb{R}$.
2. Any open interval is an open subset of $\mathbb{R}$ regardless of the topology on $\mathbb{R}$.
3. Any closed interval is a closed subset of $\mathbb{R}$ regardless of the topology on $\mathbb{R}$.
4. A half-open interval of the form $[a, b)$ is neither an open set nor a closed set regardless of the topology on $\mathbb{R}$.
5. If A is a subset of a topological space, then $A \subseteq \mathrm{Cl}(A)$.
6. If A is a subset of a topological space, then $A' \subseteq A$.
7. For any closed subset A of a topological space, $A' \subseteq A$.
8. If A is a subset of a topological space, then $\mathrm{Int}(A) \subseteq A$.
9. For any subset A of a topological space, $\mathrm{Bd}(A) \subseteq A$.
10. If A is a subset of a topological space, then $\mathrm{Bd}(A) \subseteq \mathrm{Cl}(A)$.
11. If A is a closed subset of a topological space, then $\mathrm{Bd}(A) \subseteq \mathrm{Cl}(A)$.
12. If A is a subset of a topological space, then $\mathrm{Int}(A) \subseteq \mathrm{Cl}(A)$.
13. The point 1 is a limit point of the set $[0, 1)$ regardless of the topology on $\mathbb{R}$.
14. The point 2 is not a limit point of the set $[0, 1)$ regardless of the topology on $\mathbb{R}$.
15. For any subset A of a topological space, $\mathrm{Ext}(A) = X - A$.
16. For any closed subset A of a topological space, $\mathrm{Ext}(A) = X - A$.
17. The collection $\mathscr{B} = \{\{x\} : x \in \mathbb{R}\}$ is a base for a topology on $\mathbb{R}$.
18. The collection $\mathscr{B} = \{\{x\} : x \in \mathbb{R}\}$ is a base for the usual topology on $\mathbb{R}$.
19. In a space $(X, \mathscr{T})$ any collection of open sets whose union equals X and that is closed under finite intersection is a base for $\mathscr{T}$.
20. There exists a topological space $(X, \mathscr{T})$ such that there is no base for $\mathscr{T}$.
21. There exists a topological space $(X, \mathscr{T})$ for which there is more than one base for $\mathscr{T}$.