Problem Set 5 **Set Theory & Topology** **Due 2/26/16**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. Prove Theorem 3.1.8: Let $(X, \mathscr{T})$ be a topological space with $A \subseteq X$ and $U \subseteq A$. Then $\mathrm{Cl}_A(U) = A \cap \mathrm{Cl}_X(U)$.
2. Prove Theorem 3.1.9: Let $(X, \mathscr{T})$ be a topological space with $A \subseteq X$ and $U \subseteq A$. Then $A \cap \mathrm{Int}_X(U) \subseteq \mathrm{Int}_A(U)$.
3. Prove Theorem 3.1.11: Let $(X, \mathscr{T})$ be a topological space with $A \subseteq X$ and $U \subseteq A$. Then $\mathrm{Bd}_A(U) \subseteq A \cap \mathrm{Bd}_X(U)$.
4. [Baker 3.2.12] Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces and let $A \subseteq Y$. Let $f : X \to A$ be a function. Prove that f is $\mathscr{T} - \mathscr{S}_A$ continuous iff f is $\mathscr{T} - \mathscr{S}$ continuous as a function from X to Y.
5. Prove Theorem 3.2.8: Let $(X, \mathscr{T})$ be a topological space and let $U \subseteq X$. Then $\mathrm{Int}(U) = \{x \in X : U \text{ is a nghb. of } x\}$.
6. [Baker 3.3.9] Let (a, b) and (c, d) be open intervals. Prove that the spaces $((a, b), \mathscr{U}(a, b))$ and $((c, d), \mathscr{U}(c, d))$ are homeomorphic. Don't blow off the details.
7. [Baker 3.R.7] Every constant function is continuous regardless of the topologies on the domain and codomain.
8. [Baker 3.R.8] The identity function is always continuous regardless of the topologies on the domain and codomain.
9. [Baker 3.R.9] If a function $f : \mathbb{R} \to \mathbb{R}$ is $\mathscr{U} - \mathscr{U}$ continuous, then f is $\mathscr{H} - \mathscr{U}$ continuous.
10. [Baker 3.R.10] If a function $f : \mathbb{R} \to \mathbb{R}$ is $\mathscr{U} - \mathscr{H}$ continuous, then f is $\mathscr{U} - \mathscr{U}$ continuous.
11. [Baker 3.R.11] If a function $f : \mathbb{R} \to \mathbb{R}$ is $\mathscr{H} - \mathscr{U}$ continuous, then f is $\mathscr{U} - \mathscr{U}$ continuous.
12. [Baker 3.R.12] If a function $f : \mathbb{R} \to \mathbb{R}$ is $\mathscr{C} - \mathscr{U}$ continuous, then f is $\mathscr{U} - \mathscr{U}$ continuous.
13. [Baker 3.R.13] Any two discrete topological spaces are homeomorphic.
14. [Baker 3.R.14] Any one-to-one, onto function between two discrete topological spaces is a homeomorphism.
15. [Baker 3.R.15] If f is a homeomorphism, then f is one-to-one and onto.

16. [Baker 3.R.16] If f is a one-to-one function from one topological space onto another, then f is a homeomorphism.

17. [Baker 3.R.17] If $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ are homeomorphic topological spaces, then any one-to-one function from X onto Y is a homeomorphism.

18. [Baker 3.R.18] If A and B are subspaces of $\mathbb{R}^2$, where $\mathbb{R}^2$ has the usual topology, and A can be deformed into B by use of only elastic motions, then A is homeomorphic to B.

19. [Baker 3.R.19] If A and B are subspaces of $\mathbb{R}^2$, where $\mathbb{R}^2$ has the usual topology, and A can be deformed into B by "cutting", then A is homeomorphic to B.

20. [Baker 3.R.20] If A and B are subspaces of $\mathbb{R}^2$, where $\mathbb{R}^2$ has the usual topology, and A can be deformed into B by use of elastic motions and/or "cutting", then A is homeomorphic to B.

21. [Baker 3.R.21] If A and B are subspaces of $\mathbb{R}^2$, where $\mathbb{R}^2$ has the usual topology, and A can be deformed into B by use of elastic motions and/or "cutting" and repairing the cut, then A is homeomorphic to B.

22. [Baker 3.R.22] If A and B are subspaces of $\mathbb{R}^2$, where $\mathbb{R}^2$ has the usual topology, and A can be deformed into B by use of elastic motions and/or "cutting" and then "repairing" the cut so that points close to one another before the cut are also close to one another after the cut is "repaired", then A is homeomorphic to B.