Problem Set 7 **Set Theory & Topology** **Due 4/4/16**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit.

1. [Baker 5.R.21] If $\mathbb{R}$ has the usual topology an f is a one-to-one continuous function from $\mathbb{R}$ onto $\mathbb{R}$, then f is a homeomorphism.
2. [Baker 6.1.16] Prove the Bolzano-Weierstrass Theorem (Theorem 6.1.22).
3. [Baker 6.1.17] For both the topologies $\mathscr{H}$ and $\mathscr{C}$, describe the bounded infinite subsets of $\mathbb{R}$ which do not have limit points.
4. [Baker 6.1.18] Give an example of each of the following:
   (a) a topology $\mathscr{T}$ on $\mathbb{R}$ for which there is a closed and bounded subset A of $\mathbb{R}$ that is not compact.
   (b) a topology $\mathscr{S}$ on $\mathbb{R}$ for which there is a compact subset A of $\mathbb{R}$ that is neither closed nor bounded.
5. [Baker 6.1.19] Prove that if $(X, \mathscr{T})$ is a compact topological space and $\mathscr{S}$ is any topology on X that is coarser than $\mathscr{T}$, then $(X, \mathscr{S})$ is compact.
6. [Baker 6.1.20] Give an example to show that a set X can have topologies $\mathscr{T}$ and $\mathscr{F}$ with $\mathscr{F}$ finer than $\mathscr{T}$, $(X, \mathscr{T})$ compact, and $(X, \mathscr{F})$ not compact.
7. [Baker 6.2.11] Complete the proof of Theorem 6.2.16.
8. [Baker 6.2.12] Prove Theorem 6.2.19.
9. [Baker 7.1.12] Show that the product of T_0 spaces is T_0.
10. [Baker 7.1.12] Let X be a T_1 space with $A \subseteq X$. Prove that if x is a limit point of A and U is an open set containing x, the $U \cap A$ is an infinite set.
11. [Baker 7.1.14] Let X and Y be topological spaces and let $f : X \to Y$ be a function. The *graph* of f is the set $G(f) = \{(x, y) \in X \times Y : y = f(x)\}$. Prove that if f is continuous and Y is a T_2-space, then $G(f)$ is a closed subset of the product space $X \times Y$.