Problem Set 4 **Set Theory & Topology** **Due 2/24/20**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Five of these problems will be selected (by Jon) for grading, with each worth 4 points.

1. [Baker Th 3.1.7] Let $(X, \mathscr{T})$ be a topological space with $A \subseteq X$ and $U \subseteq A$. The set U is $\mathscr{T}_A$-closed iff $U = W \cap A$ for some $\mathscr{T}$-closed set W.

2. [Baker Th 3.1.14] Let A be a subset of the space $(X, \mathscr{T})$. The set A is $\mathscr{T}$-open iff $\mathscr{T}_A \subseteq \mathscr{T}$.

3. [Baker Th 3.1.13] If $\mathscr{B}$ is a base for a topological space $(X, \mathscr{T})$ and $A \subseteq X$, then the collection $\{B \cap A : B \in \mathscr{B}\}$ is a base for $(A, \mathscr{T}_A)$.

4. [Baker Th 3.2.14] Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces and let $A \subseteq X$. If $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ is continuous, then $f|_A : (A, \mathscr{T}_A) \to (Y, \mathscr{S})$ is continuous.

5. [Baker 3.3.9] Let (a, b) and (c, d) be open intervals. Prove that the spaces $((a, b), \mathscr{U}_{(a,b)})$ and $((c, d), \mathscr{U}_{(c,d)})$ are homeomorphic.

6. Partition the spaces 1 2 3 4 5 6 7 8 9 0 into mutually disjoint collections of hemeomorphic spaces such that, if two spaces belong to different collections, then they are not homeomorphic.

7. Partition the spaces a b c d e f g h i j k l m n o p q r s t u v w x y z into mutually disjoint collections of hemeomorphic spaces such that, if two spaces belong to different collections, then they are not homeomorphic.

8. [Baker Th 4.1.10] Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces. If A and B are closed subsets of X and Y, respectively, then $A \times B$ is a closed subset of $X \times Y$.